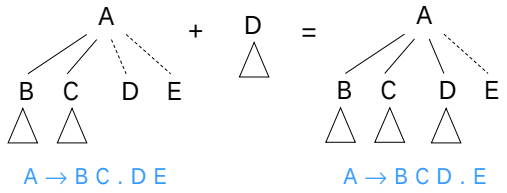
Earley's Algorithm (1970)

Nice combo of our parsing ideas so far:

- no restrictions on the form of the grammar:
  - A → B C spoon D x
- incremental parsing (left to right, like humans)
- left context constrains parsing of subsequent words
  - so waste less time building impossible things
  - makes it faster than $O(n^3)$ for many grammars

Overview of Earley's Algorithm

- Finds constituents and partial constituents in input
  - A → B C . D E is partial: only the first half of the A



A → B C . D E A → B C D . E

Overview of Earley's Algorithm

- Proceeds incrementally, left-to-right
  - Before it reads word 5, it has already built all hypotheses that are consistent with first 4 words
  - Reads word 5 & attaches it to immediately preceding hypotheses. Might yield new constituents that are then attached to hypotheses immediately preceding *them ...*
  - E.g., attaching D to A → B C . D E gives A → B C D . E
  - Attaching E to that gives A → B C D E .
  - Now we have a complete A that we can attach to hypotheses immediately preceding the A, etc.

Our Usual Example Grammar

ROOT → S			
S → NP VP		NP → Papa	
NP → Det N		N → caviar	
NP → NP PP		N → spoon	
VP → VP PP		V → ate	
VP → V NP		P → with	
PP → P NP		Det → the	
		Det → a	

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT → S	VP → VP PP	NP → Papa	V → ate
S → NP VP	VP → V NP	N → caviar	P → with
NP → Det N	PP → P NP	N → spoon	Det → the
NP → NP PP			Det → a

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1
    - 1 VP → . VP PP 1
      - 1 VP → . VP PP 1
        - 1 VP → . VP PP 1
          - 1 VP → . VP PP 1

"goal stack"

oops, stack overflowed

- OK, let's pretend that didn't happen.
- Let's suppose we didn't see VP → VP PP, and used VP → V NP instead.

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT → S	VP → V NP	NP → Papa	V → ate
S → NP VP	VP → VP PP	N → caviar	P → with
NP → Det N	PP → P NP	N → spoon	Det → the
NP → NP PP			Det → a

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1 — after dot = nonterminal, so recursively look for it ("**predict**")
    - 1 VP → . V NP 1 — after dot = nonterminal, so recursively look for it ("**predict**")
      - 1 V → . ate 1 — after dot = terminal, so look for it in the input ("**scan**")
      - 1 V → ate . 2 — after dot = nothing, so parent's subgoal is completed ("**attach**")
    - 1 VP → V . NP 2 — **predict** (next subgoal)
      - 2 NP → . ... 2 — do some more parsing and eventually ...
      - 2 NP → ... . 7 — we complete the parent's NP subgoal, so **attach**
    - 1 VP → V NP . 7 — **attach** again
  - 0 S → NP VP . 7 — **attach** again

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT	→ S	VP	→ V NP	NP	→ Papa	V	→ ate
S	→ NP VP	VP	→ VP PP	N	→ caviar	P	→ with
NP	→ Det N	PP	→ P NP	N	→ spoon	Det	→ the
NP	→ NP PP					Det	→ a

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1
    - 1 VP → . V NP 1
      - 1 V → . ate 1
      - 1 V → ate . 2
    - 1 VP → V . NP 2
      - 2 NP → . … 2
      - 2 NP → … . 7
    - 1 VP → V NP . 7

implement by function calls:
S() calls NP() and VP(), which recurse

must backtrack to try predicting a different VP rule here instead

But how about the other parse?

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT	→ S	VP	→ V NP	NP	→ Papa	V	→ ate
S	→ NP VP	VP	→ VP PP	N	→ caviar	P	→ with
NP	→ Det N	PP	→ P NP	N	→ spoon	Det	→ the
NP	→ NP PP					Det	→ a

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1
    - 1 VP → . V NP 1
      - 1 V → . ate 1
      - 1 V → ate . 2
    - 1 VP → V . NP 2
      - 2 NP → . … 2
      - 2 NP → … . 7
    - 1 VP → V NP . 7

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1
    - 1 VP → . VP PP 1
      - 1 VP → . V NP 1
        - 1 V → . ate 1
        - 1 V → ate . 2
      - 1 VP → V . NP 2
        - 2 NP → . … 2
        - 2 NP → … . 4

we'd better backtrack here too! (why?)

do some more parsing and eventually …
… the correct NP is from 2 to 4 this time
(but might we find the one from 2 to 7 instead?)

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT	→ S	VP	→ V NP	NP	→ Papa	V	→ ate
S	→ NP VP	VP	→ VP PP	N	→ caviar	P	→ with
NP	→ Det N	PP	→ P NP	N	→ spoon	Det	→ the
NP	→ NP PP					Det	→ a

- 0 ROOT → . S 0
  - 0 S → . NP VP 0
    - 0 NP → . Papa 0
    - 0 NP → Papa . 1
  - 0 S → NP . VP 1
    - 1 VP → . VP PP 1
      - 1 VP → . VP PP 1
        - 1 VP → . VP PP 1
          - 1 VP → . VP PP 1
            - 1 VP → . VP PP 1

oops, stack overflowed
no fix after all
– must transform grammar to eliminate left-recursive rules

Use a Parse Table

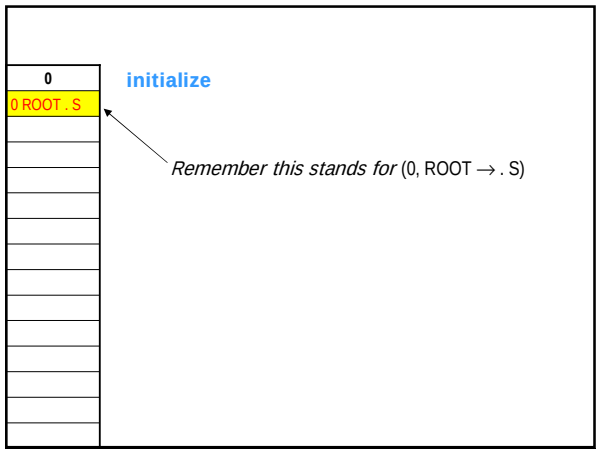
- Earley's algorithm resembles recursive descent, but solves the left-recursion problem. No recursive function calls.
- Use a parse table as we did in CKY, so we can look up anything we've discovered so far.
  "Dynamic programming."
- Entries in column 5 look like (3, S → NP . VP)
  (but we'll omit the → etc. to save space)
  - Built while processing word 5
  - Means that the input substring from 3 to 5 matches the initial NP portion of a **S → NP VP** rule
  - Dot shows how much we've matched as of column 5
  - Perfectly fine to have entries like **(3, S → is it . true that S)**

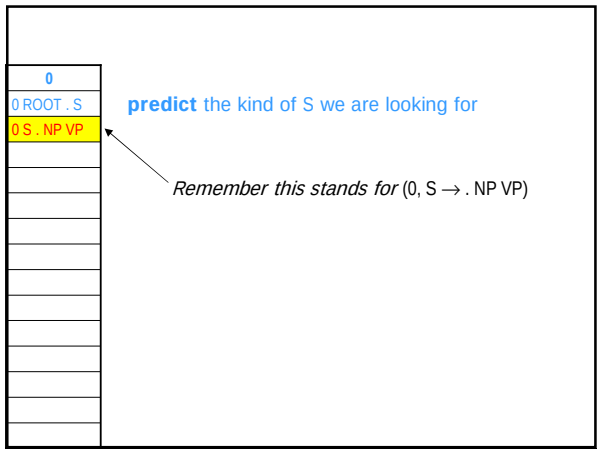
Use a Parse Table

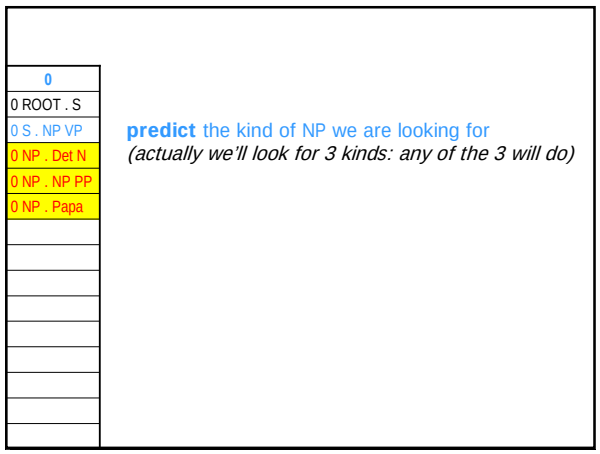
- Entries in column 5 look like (3, S → NP . VP)
- What does it mean if we have this entry?
  - *Unknown right context: Doesn't* mean we'll necessarily be able to find a VP starting at column 5 to complete the S.
  - *Known left context: Does* mean that some dotted rule back in column 3 is looking for an S that starts at 3.
    - So if we actually do find a VP starting at column 5, allowing us to complete the S, then we'll be able to attach the S to something.
    - And when that something is complete, it too will have a customer to *its* left … just as in recursive descent!
    - In short, a top-down (i.e., goal-directed) parser: it chooses to start building a constituent not because of the input but because that's what the left context needs. In **the spoon**, won't build **spoon** as a verb because there's no way to use a verb there.
    - So any hypothesis in column 5 *could* get used in the correct parse, if words 1-5 are continued in just the right way by words 6-n.

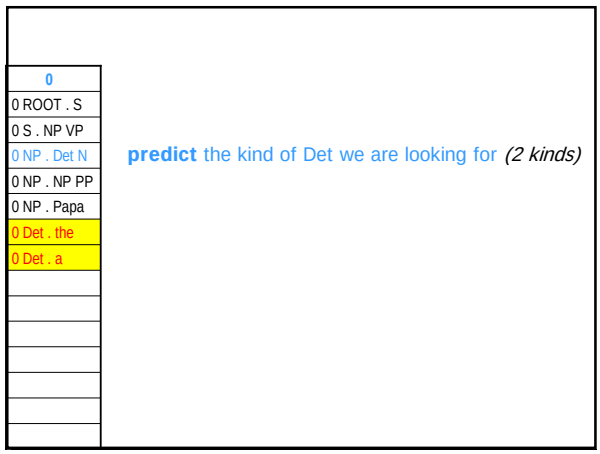
Operation of the Algorithm

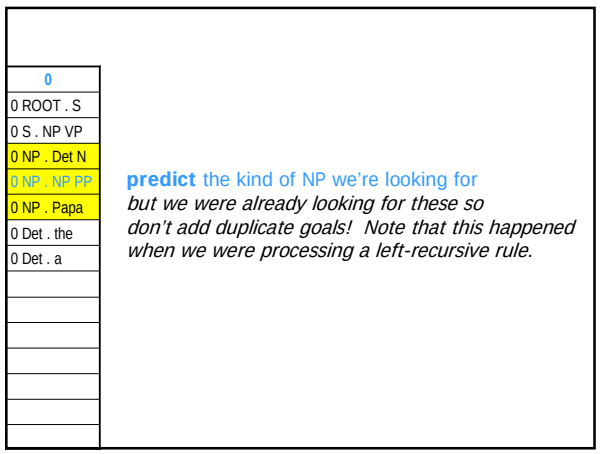
- Process all hypotheses one at a time in order. (Current hypothesis is shown in blue.)
- This may add new hypotheses to the end of the to-do list, or try to add old hypotheses again.
- Process a hypothesis according to what follows the dot – just as in recursive descent:
  - If a word, **scan** input and see if it matches
  - If a nonterminal, **predict** ways to match it
    (we'll predict blindly, but could reduce # of predictions by *looking ahead* k symbols in the input and only making predictions that are compatible with this limited *right context*)
  - If nothing, then we have a complete constituent, so **attach** it to all its customers

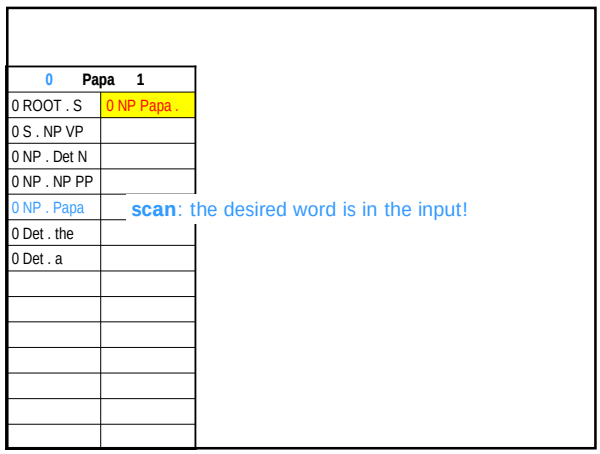

0
0 ROOT . S
initialize
Remember this stands for (0, ROOT → . S)


0
0 ROOT . S
0 S . NP VP
predict the kind of S we are looking for
Remember this stands for (0, S → . NP VP)


0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
predict the kind of NP we are looking for
(actually we'll look for 3 kinds: any of the 3 will do)


0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a
predict the kind of Det we are looking for (2 kinds)


0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a
predict the kind of NP we're looking for
but we were already looking for these so
don't add duplicate goals! Note that this happened
when we were processing a left-recursive rule.


0 Papa 1
0 ROOT . S
0 NP Papa .
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
scan: the desired word is in the input!
0 Det . the
0 Det . a

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP		
0 NP . Det N		
0 NP . NP PP		
0 NP . Papa		
0 Det . the		
0 Det . a		

scan: failure

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP		
0 NP . Det N		
0 NP . NP PP		
0 NP . Papa		
0 Det . the		
0 Det . a		

scan: failure

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP		
0 NP . Papa		
0 Det . the		
0 Det . a		

attach the newly created NP (which starts at 0) to its customers (incomplete constituents that *end* at 0 and have NP after the dot)

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the		
0 Det . a		

predict

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a		

predict

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate

predict

0 Papa 1	
0 ROOT . S	0 NP Papa .
0 S . NP VP	0 S NP . VP
0 NP . Det N	0 NP NP . PP
0 NP . NP PP	1 VP . V NP
0 NP . Papa	1 VP . VP PP
0 Det . the	1 PP . P NP
0 Det . a	1 V . ate
	1 P . with

predict

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

scan: success!

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

scan: failure

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

attach

0 Papa 1 ate 2		
0 ROOT . S	0 NP Papa .	1 V ate .
0 S . NP VP	0 S NP . VP	1 VP V . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	
	1 P . with	

predict

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP	1 VP V . NP		
0 NP . Det N	0 NP NP . PP	2 NP . Det N		
0 NP . NP PP	1 VP . V NP	2 NP . NP PP		
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

predict *(these next few steps should look familiar)*

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP	1 VP V . NP		
0 NP . Det N	0 NP NP . PP	2 NP . Det N		
0 NP . NP PP	1 VP . V NP	2 NP . NP PP		
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

predict

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP	1 VP V . NP		
0 NP . Det N	0 NP NP . PP	2 NP . Det N		
0 NP . NP PP	1 VP . V NP	2 NP . NP PP		
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

scan *(this time we fail since Papa is not the next word)*

0	Papa	1	ate	2	the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .			
0 S . NP VP	0 S NP . VP	1 VP V . NP				
0 NP . Det N	0 NP NP . PP	2 NP . Det N				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa				
0 Det . the	1 PP . P NP	2 Det . the				
0 Det . a	1 V . ate	2 Det . a				
	1 P . with					

scan: success!

0	Papa	1	ate	2	the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .			
0 S . NP VP	0 S NP . VP	1 VP V . NP				
0 NP . Det N	0 NP NP . PP	2 NP . Det N				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa				
0 Det . the	1 PP . P NP	2 Det . the				
0 Det . a	1 V . ate	2 Det . a				
	1 P . with					

0	Papa	1	ate	2	the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .			
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N			
0 NP . Det N	0 NP NP . PP	2 NP . Det N				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa				
0 Det . the	1 PP . P NP	2 Det . the				
0 Det . a	1 V . ate	2 Det . a				
	1 P . with					

0 Papa	1 ate	2 the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon
0 NP . Papa	1 VP . VP PP	2 NP . Papa	
0 Det . the	1 PP . P NP	2 Det . the	
0 Det . a	1 V . ate	2 Det . a	
	1 P . with		

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

0 Papa	1 ate	2 the	3 caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

0 Papa	1 ate	2 the	3 caviar	4	
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	**attach**
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar		
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon		
0 NP . Papa	1 VP . VP PP	2 NP . Papa			
0 Det . the	1 PP . P NP	2 Det . the			
0 Det . a	1 V . ate	2 Det . a			
	1 P . with				

0 Papa	1 ate	2 the	3 caviar	4	
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	**attach** *(again!)*
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa			
0 Det . the	1 PP . P NP	2 Det . the			
0 Det . a	1 V . ate	2 Det . a			
	1 P . with				

0 Papa	1 ate	2 the	3 caviar	4	
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	**attach** *(again!)*
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a			
	1 P . with				

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP				
	1 P . with							

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP				
	1 P . with			0 ROOT S .				

attach
(again!)

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP				
	1 P . with			0 ROOT S .				
				4 P . with				

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP				
	1 P . with			0 ROOT S .				
				4 P . with				

0	Papa	1	ate	2	the	3	caviar	4	with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .					
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .						
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .						
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP						
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .						
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP						
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP						
	1 P . with			0 ROOT S .						
				4 P . with						

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

0 Papa	1 ate	2 the	3 caviar	4 with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a
	1 P . with			0 ROOT S .	
				4 P . with	

ate	2	the	3	caviar	4	with	5	a	6
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .				
)	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP					
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N					
)	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP					
P	2 NP . Papa		0 S NP VP .	5 NP . Papa					
)	2 Det . the		1 VP VP . PP	5 Det . the					
	2 Det . a		4 PP . P NP	5 Det . a					
			0 ROOT S .						
			4 P . with						

ate	2	the	3	caviar	4	with	5	a	6
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .				
)	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N				
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N					
)	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP					
P	2 NP . Papa		0 S NP VP .	5 NP . Papa					
)	2 Det . the		1 VP VP . PP	5 Det . the					
	2 Det . a		4 PP . P NP	5 Det . a					
			0 ROOT S .						
			4 P . with						

ate	2	the	3	caviar	4	with	5	a	6
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .				
)	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N				
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar				
)	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon				
P	2 NP . Papa		0 S NP VP .	5 NP . Papa					
)	2 Det . the		1 VP VP . PP	5 Det . the					
	2 Det . a		4 PP . P NP	5 Det . a					
			0 ROOT S .						
			4 P . with						

ate	2	the	3	caviar	4	with	5	a	6
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .				
)	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N				
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar				
)	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon				
P	2 NP . Papa		0 S NP VP .	5 NP . Papa					
)	2 Det . the		1 VP VP . PP	5 Det . the					
	2 Det . a		4 PP . P NP	5 Det . a					
			0 ROOT S .						
			4 P . with						

ate	2	the	3	caviar	4	with	5	a	6	spoon	7
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .						
)	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N						
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar						
)	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP							
P	2 NP . Papa		0 S NP VP .	5 NP . Papa							
)	2 Det . the		1 VP VP . PP	5 Det . the							
	2 Det . a		4 PP . P NP	5 Det . a							
			0 ROOT S .								
			4 P . with								

ate	2	the	3	caviar	4	with	5	a	6	spoon	7
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .						
)	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP							
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N							
)	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP							
P	2 NP . Papa		0 S NP VP .	5 NP . Papa							
)	2 Det . the		1 VP VP . PP	5 Det . the							
	2 Det . a		4 PP . P NP	5 Det . a							
			0 ROOT S .								
			4 P . with								

ate	2 the	3 caviar	4 with	5 a	6 spoon	7
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .
P	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	5 NP Det N .
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	4 PP P NP .
P	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	5 NP NP . PP
P	2 NP . Papa		0 S NP VP .	5 NP . Papa		
P	2 Det . the		1 VP VP . PP	5 Det . the		
	2 Det . a		4 PP . P NP	5 Det . a		
			0 ROOT S .			
			4 P . with			

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		
	1 P . with			0 ROOT S .		
				4 P . with		

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		
				4 P . with		

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP

0 Papa	1 ate	2 the	3 caviar	4 with a spoon 7		
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon		7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon		7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with

0 Papa	1 ate	2 the	3 caviar	4 with a spoon		7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4 with a spoon		7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4 with a spoon		7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

0 Papa	1 ate	2 the	3 caviar	4 with a spoon		7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

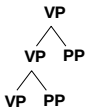
0 Papa	1 ate	2 the	3 caviar	4 with a spoon	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	...	6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP
	1 P . with			0 ROOT S .		1 VP V NP .
				4 P . with		2 NP NP . PP
						0 S NP VP .
						1 VP VP . PP
						7 P . with
						0 ROOT S .

Left Recursion Kills Pure Top-Down Parsing ...

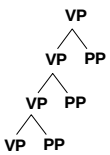
VP

Left Recursion Kills Pure Top-Down Parsing ...

Left Recursion Kills Pure Top-Down Parsing ...



Left Recursion Kills Pure Top-Down Parsing ...



makes new hypotheses ad infinitum before we've seen the PPs at all

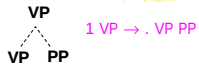
hypotheses try to predict in advance how many PP's will arrive in input

... but Earley's Alg is Okay!

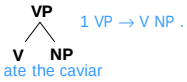
1 VP → . VP PP

(in column 1)

... but Earley's Alg is Okay!

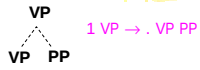


(in column 1)



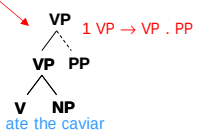
(in column 4)

... but Earley's Alg is Okay!



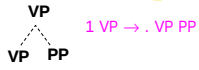
(in column 1)

attach

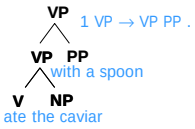


(in column 4)

... but Earley's Alg is Okay!

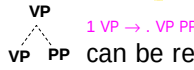


(in column 1)



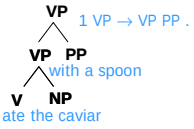
(in column 7)

... but Earley's Alg is Okay!



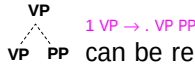
can be reused

(in column 1)



(in column 7)

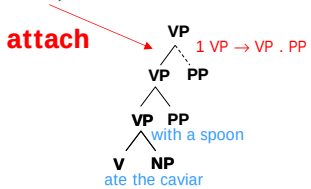
... but Earley's Alg is Okay!



can be reused

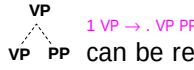
(in column 1)

attach



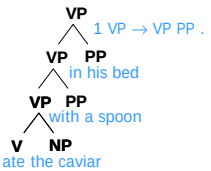
(in column 7)

... but Earley's Alg is Okay!



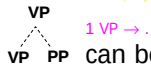
can be reused

(in column 1)



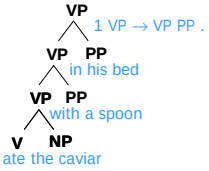
(in column 10)

... but Earley's Alg is Okay!



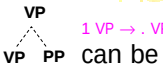
1 VP → . VP PP can be reused again

(in column 1)



(in column 10)

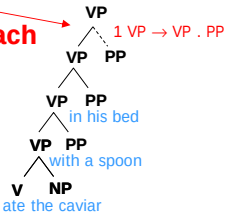
... but Earley's Alg is Okay!



1 VP → . VP PP can be reused again

(in column 1)

attach



(in column 10)

0 Papa	1 ate	2 the	3 caviar	4 with a spoon	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	... 6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP NP . PP
0 NP . Papa	**1 VP . VP PP**	2 NP . Papa		0 S NP VP .	2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		**1 VP VP . PP**	1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	7 PP . P NP
	1 P . with			0 ROOT S .	1 VP V NP .
				4 P . with	2 NP NP . PP
					0 S NP VP .
					1 VP VP . PP
					7 P . with
					0 ROOT S .

completed a VP in col 4
col 1 lets us use it in a VP PP structure

0 Papa	1 ate	2 the	3 caviar	4 with a spoon	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	... 6 N spoon .
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	5 NP Det N .
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	4 PP P NP .
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP NP . PP
0 NP . Papa	**1 VP . VP PP**	2 NP . Papa		0 S NP VP .	2 NP NP PP .
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	1 VP VP PP .
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	7 PP . P NP
	1 P . with			0 ROOT S .	1 VP V NP .
				4 P . with	2 NP NP . PP
					0 S NP VP .
					1 VP VP . PP
					7 P . with
					0 ROOT S .

completed that VP = VP PP in col 7
col 1 would let us use *it* in a VP PP structure
can reuse col 1 as often as we need

What's the Complexity?