Binaural dissimilarity and optimum ceilings for concert halls: More lateral sound diffusion

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Ceiling shapes for concert halls are proposed that, on the basis of prior extensive subjective evaluations, should result in high listener preference of the hall's acpistoca; response to music. These shapes are based on the premise that as little as possible of the early sound energy should arrive at a listener's ears in the "median" plane (the vertical symmetry plane through the listener). While this goal is inherently approached in old-style, high-and-narrow halls, its realization in modern, low-ceiling halls requires special ceiling shapes and surface structures to keep early, median-plane sound away from the listener's ears.

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I. BINAURAL DISSIMILARITY

In an extensive subjective evaluation of European concert halls, Schroeder, Gottlob, and Siebrasse[1] found that the "binaural dissimilarity" of the two ear signals, recorded from a (specially designed) dummy head was highly correlated with the subjective preference data. Specifically, for concert halls and seat positions with highly similar ear signals, the subjective preference was low. Conversely, for more dissimilar ear signals the preference was high. In other words, binaural dissimilarity and subjective preference are positively correlated. In fact, binaural dissimilarity was found to be at least as strongly correlated with preference as any other objective parameter—including reverberation time.

The musical program material for these studies was from the classical and romantic repertoire. However, considering the fundamental invariance of all musical perception—as opposed to speech perception—one may expect corresponding results for other musical styles as well.

Binaural similarity (called "interaural coherence" and abbreviated "C" in Ref. 1) is defined as the peak value of the correlation function of the first 80 ms of the impulse responses within an interaural delay range of 1 ms. Binaural *dis*similarity is defined as the negative of binaural similarity.

The results imply that sound arriving in the median plane of a listener is detrimental to subjective preference because such median-plane sounds lead to identical sound pressure waves at the two ears thereby decreasing binaural dissimilarity. (An explanation for the detrimental effect of median-plane sound is that it leads to a more "monophonic"—as opposed to "stereophonic"—listening experience.[2-5])

This conclusion is confirmed by another result of the investigation: The highest correlation between subjective preference and a geometrical parameter of the hall was with the *width* of the hall; the narrower the hall, the higher the preference and vice versa. Narrow halls, of course, have more numerous and more powerful early lateral reflections that arrive at the ears near the horizontal plane as opposed to the (vertical) median plane. Such early lateral reflections lead to a greater binaural dissimiarity, and therefore higher listener preference. Others who have discussed the effects of early lateral reflections include West,[6] Marshall,[7,8] Keet,[9] Barron[10] and Kuhl.[11]

More recently, even more direct evidence for the importance of early lateral reflections has been obtained by the author and his collaborators at Göttingen.[12-16] While in the study of real halls different physical parameters come in a "predetermined mix," which can only be disentangled *statistically* by appropriate subjective scaling methods, the technique of digital simulation[17] can be employed to alter *one* physical parameter at a time and to study its effect on musical preference independently of all other potentially confusing factors.

Accordingly, binaural impulse responses from real halls were fed into a computer and digitally modified by adding (or deleting) lateral reflections. When the effect of these modifications is expressed as binaural similarity (as defined above), it was found that subjective preference reached its maximum value for zero binaural similarity.[12] The lowest subjective preference score was obtained for the highest value of binaural similarity. Interestingly, low preference was also found for negatively correlated ear signals. Thus, within the scope of these investigations, it can be concluded that zero binaural similarity is optimum even if in actual enclosures this goal cannot be completely realized.

II. CEILING SHAPES

How then can we design concert hall ceilings that avoid direct (median-plane) reflections into the audience area? Absorption is *verboten* because we need the energy from the ceiling for reverberation. However, we can shape the ceiling to reflect most of the sound energy to the *side walls*, whence it will reach the listener, directly or indirectly, from lateral directions.

One possible ceiling shape would be a convex (curved downward over the "center aisle") hyperbolic cylinder whose axis runs parallel to the front-back axis of the hall. A hyperbolic surface, as is well known, reflects

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rays from one of its focal lines as if they were coming from the other focal line. Thus, if the outer focal line of the ceiling's hyperbolic cross section is located in the stage area at the height of the musical instruments, the reflected sound rays will seem to come from a point only slightly above the ceiling and will spread out accordingly with much more energy going laterally than for a plane ceiling.

More generally and less mathematically speaking, the ceiling should droop above the center aisle with a U-shaped cross section to minimize median plane reflections.

III. HIGHLY SOUND-DIFFUSING CEILINGS

Another and possibly more effective solution would be to design a celing with extremely diffuse sound reflection, scattering a simple ray into ten or more "ray-lets" with roughly equal energies over a wide angular distribution. Surfaces with such highly diffuse reflections including experimental results were described previously.[18-21] These surfaces were designed to have local reflection coefficients for normally incident sound alternating between +1 and −1 according to a mathematical sequence (binary "maximum-length sequence," also called binary "pseudorandom noise"). Reflection coefficients of −1 for hard surfaces are easily realized by a quarter-wavelength "wells" in the wall.

However, because of the quarter wavelength requirement, the sound diffusing properties of the surface depend upon the wavelength of the incident sound. In practice it is found that good diffusion is obtained in a range of wavelengths half an octave below and above the "design wavelength." For an incident wave of half the design wavelength, the wells are, of course, half a wavelength deep, resulting in near *specular* reflection by the surface. (However, at one-third the design wavelength, the surface is a good diffusor again).

IV. QUADRATIC-RESIDUE DIFFUSORS

In the meantime, we have looked hard for surface structures that give excellent sound diffusion over larger wavelength ranges. In this search, we discovered, by computer simulation, that surfaces based on m-ary maximum-length sequences[22] are capable of good diffusion over larger bandwidths—presumably because such surfaces have wells of several different depths. This led us to still another kind of mathematical sequences, the quadratic-residue sequences of elementary number theory, investigated by A. M. Legendre and C. F. Gauss. These sequences are defined as follows

$$s_n = n^2 \text{ (where } n^2 \text{ is taken as the least non-negative remainder nodulo } N, \text{ and } N \text{ is an odd prime).} \tag{1}$$

As an example, for $N=17$, the sequence reads (starting with $n=0$):

0, 1, 4, 9, 16, 8, 2, 15, 13, 13, 15, 2, 8, 16, 9, 4, 1; 0, 1, 4, etc.

These sequences are symmetric [around $n \equiv 0$ and $n \equiv (N-1)/2$] and periodic with period N. In addition, they have the following astounding property: The discrete Fourier Transform R_m of the exponentiated sequence

$$r_n = \exp(i2\pi s_n/N), \tag{2}$$

has constant magnitude. In fact,

$$|R_m|^2 = |(1/N)\sum_{n=1}^{N} r_n \exp(-i2\pi nm/N)|^2 = 1/N. \tag{3}$$

Of the several proofs of this relationship, a particularly simple one is based on the fact that the autocorrelation sequence of r_n is zero except for a zero shift (modulo N).[23] And this, in turn, is a consequence of the fact that the s_n form a complete difference set, i.e., that no matter what interval k one considers, the pairwise differences $s_{n+k} - s_n$ (mod N) generate all the numbers between 0 and $N-1$ exactly once per period. Thus, for $k=1$, for example, the differences generated are:

1, 3, 5, 7, 9, 11, 13, 15, 0, 2, 4, 6, 8, 10, 12, 14, 16.

As a consequence, the autocorrelation becomes the sum of a *complete set of roots of unity* which is, of course, zero. As S. W. Golomb[22] has shown, the spectrum of a sequence that has a two-valued autocorrelation is flat.

Now suppose we construct a hard wall and give it local reflection coefficients for normal incidence according to the exponentiated quadratic-residue sequence r_n.[24] (See Fig. 1 for an illustration of a cross section through such a surface for $N=17$.) What kind of sound reflection properties might it have? This is a complicated diffraction problem. Nevertheless, by making several, rather bold, simplifications, we can get at least some approximate answers.

We shall consider the reflecting surface as planar with a local impedance $Z(x)$ which varies only along one of its dimensions (x) in a periodic fashion with period Nw (see Fig. 1). $Z(x)$ is constant in the direction orthogonal to x. Thus, we can treat the diffraction problem as a two-dimensional problem, the two dimensions being the x direction and the direction normal to the reflecting surface. By definition:

$$Z(x) = p(x)/v(x), \tag{4}$$

where $p(x)$ is the sound pressure and $v(x)$ is the component of the particle velocity normal to the reflecting surface (the normal being directed *into* the surface).

The sound pressure $p(x)$ is composed of that of the incident or "arriving" wave $p_a(x)$ and the scattered waves $p_s(x)$:

$$p(x) = p_a(x) + \sum_s p_s(x). \tag{5}$$

If the incident wave has unit amplitude and an angle of incidence α_a with respect to the surface normal, one has

$$p_a(x) = \exp(-ik_a x), \quad \text{with } k_a = (2\pi/\lambda)\sin\alpha_a. \tag{6}$$

The dependence of the scattered waves on x is

$$p_s(x) = a_s \exp(-ik_s x), \quad \text{with } k_s = k_a + 2\pi s/Nw, \tag{7}$$

where Nw is the period of the impedance $Z(x)$. The a_s are the yet unknown scattering amplitudes—which we

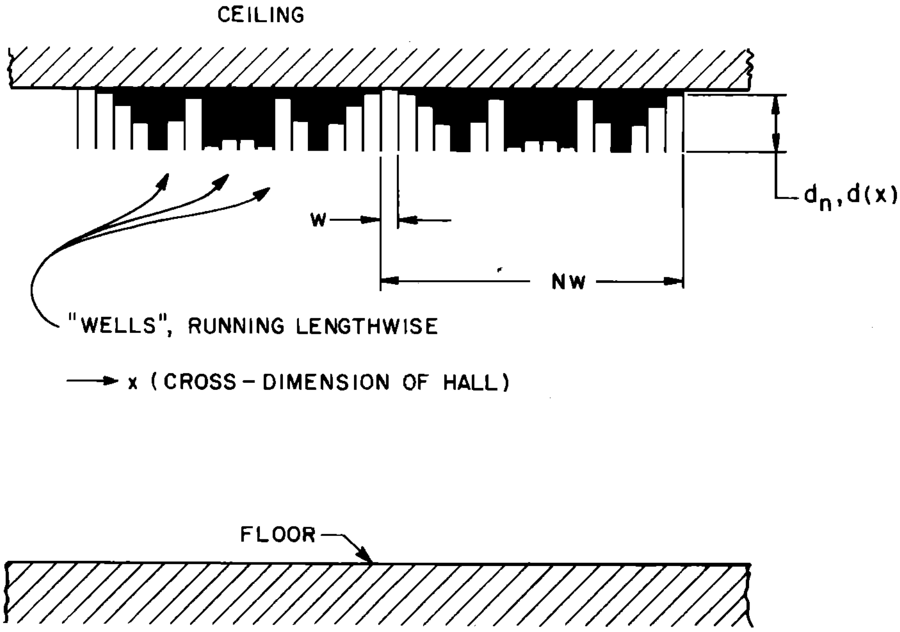

FIG. 1. Lateral cross section through diffusely reflecting ceiling based on quadratic residue sequence with $N=17$. The width of each "well" equals $0.137\lambda_0$, where λ_0 (3 cm in the scale model) is the "design" wavelength. The depths of the wells $d(x)$ vary according to Eq. (15) from 0 to a little less than half the design wavelength. The thin vertical lines represent rigid separators between individual wells. They are crucial for good diffusion, particularly for obliquely incident sound.

hope to be as uniform in magnitude as possible. (The relation for k_s, the "spatial frequencies" in the x direction, is the same as that in Ref. 18, Eqs. (2a) and (2b), except for a change in the notations.)

The normal particle velocity component $v(t)$ is obtained from the sound pressure $p(x)$ by differentiating in the direction of the normal n:

$$v(x)=(i\lambda/2\pi\rho c)\partial p(x)/\partial n\,. \tag{8}$$

Here ρc is the characteristic impedance of the medium (air) facing the reflecting surface.

With the standard wave equation for sound propagation in two dimensions, one can easily execute the operation $\partial p(x)/\partial n$, yielding

$$v(x)=i(\cos\alpha_a-\sum_s a_s[1-(k_s\lambda/2\pi)^2]^{1/2})/\rho c\,. \tag{9}$$

For values of the summation index s in the range

$$-(Nw/\lambda)(1+\sin\alpha_a)\leq s\leq(Nw/\lambda)(1-\sin\alpha_a)\,, \tag{10}$$

the square root in Eq. (9) will be real and will equal the cosine of the scattering angle: $\cos\alpha_s$. This range of s corresponds to *propagating* reflected waves. For s outside the above range, the scattered waves will be evanescent with zero energy flux. Both types of waves are important in an exact solution for the unknown scattered amplitude a_s.

However, if, in a rough approximation, one neglects the evanescent waves and furthermore sets $\cos\alpha_s=1$ for the reflected waves, he obtains from Eqs. (4)–(7) and (9).

$$Z(x)=\rho c\left[1+\sum_s p_s(x)\right]\Big/\left[\cos\alpha_a-\sum_s{}' p(x)\right]\quad, \tag{11}$$

where the $\sum'$ indicates that summation index s is restricted to propagating waves.

Equation (11) is easily solved for the reflected waves. For the case of a normally incident wave ($\alpha_a=0$) one has

$$\sum_s{}' p_s(x)=[Z(x)-\rho c]/[Z(x)+\rho c]\,, \tag{12}$$

from which the scattering amplitudes a_s are obtained by a Fourier transformation over the interval $0\leq x\leq Nw$, i.e., the period of the $p_s(x)$ [cf. Eq. (4)]:

$$a_s=\frac{1}{Nw}\int_0^{Nw}\frac{[Z(x)-\rho c]}{[Z(x)+\rho c]}\exp\frac{Nw}{i2\pi sx}\,dx\,. \tag{13}$$

Next we make the reasonable assumption that the local impedance of a surface with wells such as shown in Fig. 1 has a local impedance

$$Z(x)=\rho c/i\tan[2\pi d(x)/\lambda]\,,$$

where $d(x)$ is the depth of the well at x. Then

$$a_s=\left(\frac{1}{Nw}\right)\int_0^{Nw}\exp\left[-\frac{\lambda}{i4\pi d(x)}+\frac{Nw}{i2\pi sx}\right]dx\,. \tag{14}$$

If the $d(x)$ are constant over a width w that is small compared to the wavelength and then change abruptly to a new value (see Fig. 1) given by the quadratic-residue sequence [Eq. (1)].

$$d_n=(\lambda/2N)s_n\,, \tag{15}$$

then the Fourier integral can be approximated by a sum with the result, from Eqs. (2) and (3),

$$|a_s|^2=\text{const.}=1/N\,,$$

i.e., under the simplifying assumptions made, the amplitude of each scattered wave is the same—in a sense, the quadratic-residue surface is an optimum diffusor.

If one desires to go beyond this first-order approximation, one has to solve the equation

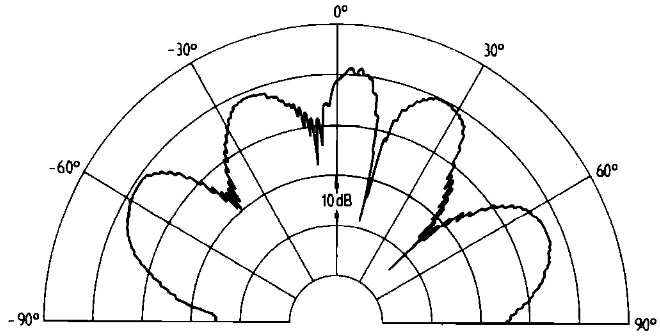

FIG. 2. Scatter diagram of the diffusor shown in Fig. 1 for the design frequency $f_0 = 11.5$ kHz. Figures 2–6 are for a normally incident sound wave.

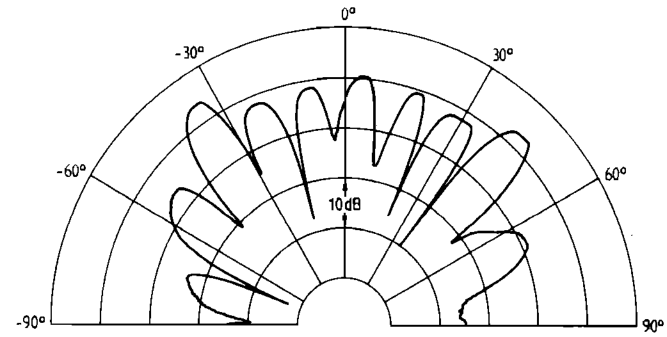

FIG. 3. Scatter diagram of the diffusor shown in Fig. 1 for $f = 2f_0 = 22.9$ kHz.

$$Z(x)v(x) = p(x) ,$$

for the a_s, with $p(x)$ as in Eqs. (5)–(7) and $v(x)$ as in Eq. (9). With $Z(x)$ periodic, this is accomplished by a Fourier transformation, which yields an infinite set of linear equations. We have found that by restricting this set to 50–100 terms (for $N = 17$), stable solutions for the a_s can be obtained on sufficiently accurate computers. All subsequent results were obtained by inverting matrices of size 99×99.

For the surface shown in Fig. 1 ($N = 17, w = 0.137\lambda_0$, where λ_0 is the "design" wavelength) and vertically incident sound, we obtained the following values for the five scattering amplitudes:

scattering angle $\alpha_s = -59°, -25°, 0°, 25°, 59°$,

scattering amplitude $|a_s| = 0.69, 0.49, 0.30.0.49, 0.69$.

Thus, the true scattering amplitudes are seen to be not as uniform as the simplified theory predicted. However, the energy fluxes, taking the projecting cosines into account, show remarkable uniformity:

$$|a_s|^2 \cos\alpha_s = 0.24, 0.21, 0.09, 0.21, 0.24 ,$$

The sum of these energy fluxes equals one (as it should for a nonabsorbing surface) with an accuracy of better than 10^{-3} if the unrounded values are taken.

The uniformity of the reflected energy fluxes can be described most effectively by the standard deviation σ in decibels of the energy fluxes. For the above case, $\sigma = 1.7$ dB. Thus, while the scattering amplitudes are not uniform, the subjectively important standard deviation of the energy fluxes expressed in decibels is rather small. (The ear, in a laboratory situation, can distinguish level differences of the order of 1 dB.)

Figure 2 shows the result of a measurement on a scale model of the surface structure at a frequency of 11.5 kHz (corresponding to the design wavelength). Both the observed scattering angles and the energy fluxes are in good agreement with the theoretical values.

What are the scattering properties of the quadratic-residue surfaces at wavelengths other than the design wavelength? For longer wavelengths, the ceiling will look more and more like a smooth mirror, and we may not expect good scattering for frequencies half an octave or more below the frequency corresponding to λ_0.

However, for wavelengths shorter than λ_0, the ceiling remains a good scatterer up to a limit given by prime number N: $\lambda > \lambda_0/N$ and, more stringently, the width w: $\lambda > 2w$.

Specifically, for frequencies that are integer multiples of c/λ_0 (where c is the velocity of sound) the "reflection coefficients" are

$$r_k(f = mc/\lambda_0) = \exp(i2\pi m k^2/N) .$$

For $m = 1, 2, \ldots, N-1$, this sequence is a permutation of the original sequence ($m = 1$) and has the same two-valued autocorrelation as the original sequence and therefore uniform sound scattering properties within the simplified theory.

Computations of the scattered energy fluxes for the surface shown in Fig. 1 for four different wavelengths give the following standard deviations

λ_0: $\sigma = 1.7$ dB, $\lambda_0/2$: $\sigma = 3.3$ dB,

$\lambda_0/3$: $\sigma = 3.7$ dB, $\lambda_0/4$: $\sigma = 2.4$ dB.

Experimental results for higher frequencies are shown in Fig. 3 (for 22.9 kHz), Fig. 4 (for 34.4 kHz), and Fig. 5 (for 45.9 kHz). As can be seen, there is still good diffusion even at the highest frequency. In fact, the higher the frequency the more scattering angles appear, in accordance with Eq. (10).

For intermediate frequencies, i.e., frequencies that

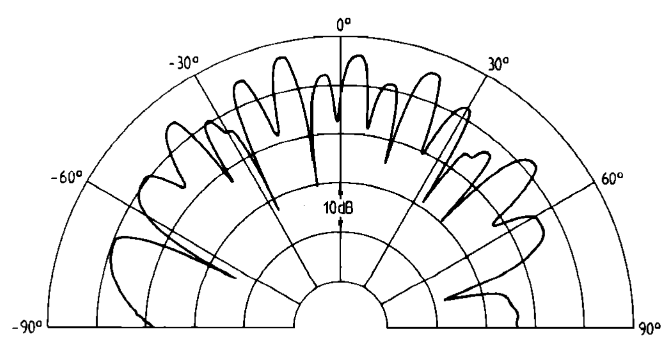

FIG. 4. Scatter diagram of the diffusor shown in Fig. 1 for $f = 3f_0 = 34.4$ kHz.

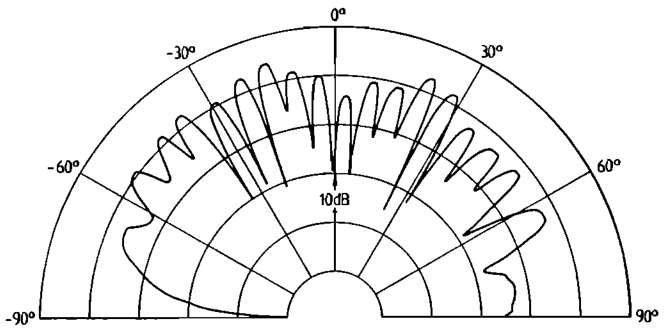

FIG. 5. Scatter diagram of the diffusor shown in Fig. 1 for $f=4f_0=45.9$ kHz.

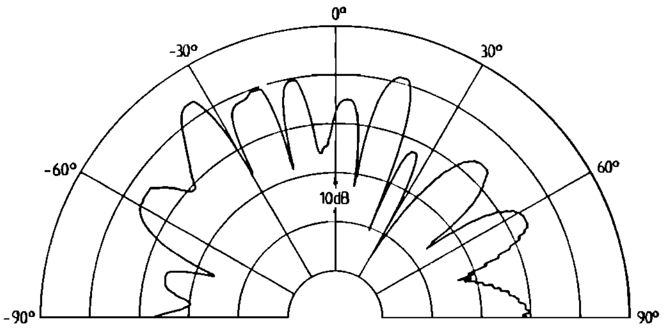

FIG. 7. Scatter diagram for diffusor shown in Fig. 1 for an obliquely ($\alpha_a = 55°$) incident sound wave for $f = 22.9$ kHz. The scattered energy covers the entire range from −90° to +90°, but is somewhat less uniformly distributed as for normal incidence at the same frequency (cf. Fig. 3).

are not integer multiples of the design frequency, similarly good results are obtained. For ten randomly selected frequencies, in the first and second octaves above the design frequencies the following average standard deviations were found computationally:

$$(\lambda_0, \lambda_0/2): \ \bar{\sigma} = 2.4 \text{ dB},$$

$$(\lambda_0/2, \lambda_0/4): \ \bar{\sigma} = 4.2 \text{ dB}.$$

Experimentally, too, good scattering is found at noninteger multiple frequencies. As a typical example, Fig. 6 shows the scatter diagram for 32.4 kHz.

Finally, even for oblique incidence the wide-angle scattering properties are well maintained. Figure 7 shows the scatter diagram for an incident angle of $\alpha_a = 55°$ and a frequency of 22.9 kHz.

How much better than *random*-surface diffusors are the quadratic residue diffusors? For four different diffusors with *randomly* selected well depths, the following standard deviations of the energy fluxes were obtained

$$(\lambda_0, \lambda_0/2): \ \bar{\sigma} = 4.8; 5.2; 4.0; 4.7 \text{ dB},$$

$$(\lambda_0/2, \lambda_0/4): \ \bar{\sigma} = 4.1; 5.6; 5.6; 5.6 \text{ dB}.$$

Thus, none of the four random diffusors is as good as the quadratic residue diffusors, *even when averages over ten randomly selected frequencies are considered*!

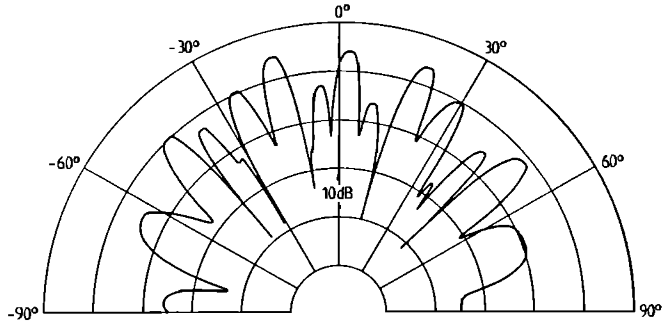

FIG. 6. Scatter diagram of the diffusor shown in Fig. 1 for a nonintegral frequency $f = 32.4$ kHz.

V. TWO-DIMENSIONAL SCATTERERS

So far one-dimensional sound scattering has been considered. In other words, if the wells on the ceiling run lengthwise, sound waves from the stage are diffused laterally from the ceiling, but are reflected mirror-like in the front–back dimension.

If we desire also longitudinal diffusion, the ceiling surface needs structuring also in the longitudinal direction. This is easily realized by replacing the quadratic residue sequence of Eq. (1), by a two-dimensional quadratic residue *array* of reflection coefficients.

$$r_{k,l} = \exp[i2\pi(k^2 + l^2)/N], \tag{16}$$

where a unit step in l (from l to $l+1$) corresponds to a distance w_l in the *front–back* direction.

Such two-dimensional diffusors were constructed on a model scale. Measurements confirmed the expected excellent diffusion over the entire solid angle. For an illustration of such a diffusor, see Fig. 8.

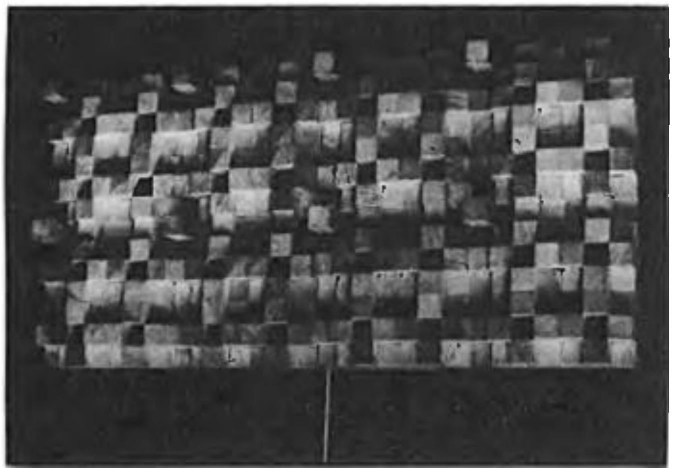

FIG. 8. Photograph of scale model of *two*-dimensional diffusor based on quadratic residue array, for efficient diffusion over the entire solid angle.

If "back scattering" (backward from the ceiling to the stage) is desired even at the highest frequency the longitudinal widths w_i should be made a quarter-wavelength rather than a half-wavelength.

If esthetically preferred, the ceiling pattern given by Eq. (16) can be rotated around a vertical axis by, say, 45° without detrimental effect on sound diffusion.

VI. COMBINATION CEILING DESIGNS

The two ceiling designs for maximizing early lateral reflections described here—one based on geometrical principles, the other on sound diffraction—can be profitably combined.

The diffraction principle allows either one or two-dimensional diffusion—with different frequency ranges, if desired.

The amount of flexibility inherent in the proposals made here should be sufficient to achieve the preponderance of lateral reflections necessary for the high listener preference found in concert halls with large binaural dissimilarity. A first application of quadratic-residue diffusors to the design of a large hall is described by Marshall and Hyde.[25]

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